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BARGAINING WITH RENEGOTIATION IN MODELS WITH ON-THE-JOB SEARCH

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This paper studies the role of partial commitment in models with on-the-job search. Commitment is modeled as the frequency at which wages are renegotiated. This formulation nests earlier models in the literature as special cases when the frequency of renegotiation goes to zero (full commitment) or infinity (no commitment). In this setup, I first show that the degree of commitment is important for the share of the surplus captured by the worker. With no commitment, the worker value reflects only her bargaining power. With commitment, the worker receives a higher share of the surplus, because a higher wage increases the total surplus by increasing the length of the match. The length of the match is more responsive to the agreed wage when commitment is higher. When the model is calibrated, the values of the model primitives, e.g. the bargaining power of workers and the productivity distribution, differs starkly depending on the assumed degree of commitment. Second, I show that when the degree of commitment is endogenous, firms will in general choose an intermediate level of commitment, rather than a corner solution. I also show that the equilibrium wage distribution and the bargaining outcomes are unique.

Bargaining with renegotiation in models with on-the-job search*

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Abstract

This paper studies the role of partial commitment in models with on-the-job search. Commitment is modeled as the frequency at which wages are renegotiated. This formulation nests earlier models in the literature as special cases when the frequency of renegotiation goes to zero (full commitment) or infinity (no commitment). In this setup, I first show that the degree of commitment is important for the share of the surplus captured by the worker. With no commitment, the worker value reflects only her bargaining power. With commitment, the worker receives a higher share of the surplus, because a higher wage increases the total surplus by increasing the length of the match. The length of the match is more responsive to the agreed wage when commitment is higher. When the model is calibrated, the values of the model primitives, e.g. the bargaining power of workers and the productivity distribution, differs starkly depending on the assumed degree of commitment. Second, I show that when the degree of commitment is endogenous, firms will in general choose an intermediate level of commitment, rather than a corner solution. I also show that the equilibrium wage distribution and the bargaining outcomes are unique.

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1 Introduction

This paper studies the role of commitment in models with on-the-job search (OJS) in which wages affect turnover. In the literature, two extreme cases of commitment have been analyzed. In wage posting models ([Burdett and Mortensen, 1998](#)) and some models with bargaining ([Mortensen, 2003](#); [Shimer, 2006](#)) agents can commit to a given wage forever. In contrast, in the model with Nash bargaining of [Pissarides \(1994\)](#) match surplus is treated as fixed in the bargaining stage, which I show corresponds to the case when the agents are unable to commit at all.

I explore the equilibrium in a model in which the agents can only commit to a wage for some period of time. I do this by setting up a model with (i) OJS, (ii) renegotiation and (iii) non-cooperative bargaining. The model extends [Shimer \(2006\)](#) to incorporate renegotiation and different bargaining powers. Wages are determined in a bargaining game with alternating offers by the firm and the worker in the spirit of [Rubinstein \(1982\)](#) and [Binmore et al. \(1986\)](#). Between offers, there is an exogenous probability that the bargaining process breaks down. In the event of a breakdown, the worker becomes unemployed and the firm's payoff is zero. If an agreement is reached, the wage remains fixed until the end of the wage contract at which time they renegotiate, i.e. bargain over a new wage. The end of the contract arrives at a Poisson rate and the Poisson intensity represents the ability of agents to commit. Job offers arrive at a Poisson rate to the employed.

Renegotiation, and thereby partial commitment, accounts for a realistic feature of labor markets, and has important implications for the equilibrium of the model. The paper makes four points: (i) the frequency of renegotiation plays a very big role in the model (e.g. in a calibrated model the primitives of the model differ starkly depending on the frequency of renegotiation) (ii) the two extremes cases of commitment analyzed in the literature, in general, do not correspond to the optimal choice by the firm, (iii) the pay-offs in previous papers with cooperative bargaining, [Pissarides \(1994\)](#) and [Mortensen \(2003\)](#), correspond to special cases of my model, and (iv) there is a unique equilibrium in the model unlike in the model of [Shimer \(2006\)](#).

An important feature of the model is that the match surplus increases when turnover decreases, and that turnover decreases if a higher wage is agreed upon. In order to understand the mechanism, it is useful to first consider how the surplus responds to turnover, and then how turnover responds to the wage. In the model, when a worker receives a job offer, she unilaterally decides whether or not to quit. When the worker quits, the profits of the current firm is lost. If the increase in worker value is less than the profits in the current firm, then the job switch is bilaterally inefficient

for the worker and the firm. This is due to the firm losing more profits than the worker gains in value (the transition might still be socially efficient if the worker moves to a more productive job). If the firm and the worker agree on a higher wage, then the worker will quit less often. Since these transitions are bilaterally inefficient, total surplus is higher if a higher wage is agreed upon. Unlike in search models without OJS, the values are not perfectly transferable. The change in surplus from a marginal change in the wage is given by the change in turnover multiplied by the level of profits (i.e. the retained value times the retention rate). Consider a small increase in the agreed wage for a particular match quality. In this case, the change in turnover is given by the product of three terms: (i) how often job offers arrive, (ii) the density of job offers at the particular wage and (iii) how the worker trades off the higher wage against being matched with a type that in equilibrium pays that wage. The third term, the marginal relative value of the wage, is given by the ratio of the expected duration of the wage over the expected duration of the match. When wages are never renegotiated as in [Mortensen \(2003\)](#) and [Shimer \(2006\)](#), then the duration of the wage and the match is the same, and the worker cares only about the wage. When wages are continuously renegotiated, the worker cares only about the match quality. In the intermediate case, the worker cares about both the match quality and the wage.

The share of the surplus captured by the worker reflect both the relative bargaining power and how surplus response to the wage. The bargaining outcome depends critically on how the match surplus responds to the wage. If the surplus increases with the wage, the worker will capture a higher share. This change in surplus is, in turn, large if wages are infrequently renegotiated. When wages are continuously renegotiated, there is the wage has no impact on the turnover of the worker, and the equilibrium can be described by the Nash bargaining solution with transferable utilities.¹ Hence, one result in the present paper is that a Nash bargaining solution with transferable values can be justified in models with OJS if there is continuous renegotiation.²

¹When firms have all the bargaining power and the wages are continuously renegotiated then the wage distribution converges to a mass point at the reservation wage which is then equal to the benefit level. This contrast with the models of [Coles \(2001\)](#) and [Coles and Mortensen \(2016\)](#). In these models, firms have all the bargaining power and can continuously adjust the wage. But in [Coles and Mortensen \(2016\)](#), the worker cannot observe the match quality. This is similar to [Burdett and Mortensen \(1998\)](#) and [Shimer \(2006\)](#) in that the worker's value function only depends on the wage, unlike my model. In the model presented in [Coles \(2001\)](#), match quality (employment level in the paper) is observable. But [Coles \(2001\)](#) consider a model in which the future wage expectations changes to the reservation wages if the firm deviates and sets a lower wage. Thus, in [Coles \(2001\)](#) and [Coles and Mortensen \(2016\)](#), but unlike this paper, the expectation of all future wages in the match change if a lower wage is set today.

²[Krause and Lubik \(2007\)](#) analyze a model in discrete time in which wages are set in the beginning of the each

The frequency of renegotiation is important for the share of the surplus that the worker captures. In order for a model with more frequent renegotiation to match the same labor share it must then be that the bargaining power of workers is higher. I illustrate this by setting the job offer arrival rates and separation rates to match transition moments from the literature. I then estimate the productivity distribution and the bargaining power of the worker in order to match a labor share of $2/3$ and a log Normal wage offer distribution (with a scale parameter from the literature). When I do this analysis, the bargaining power of workers is calibrated to be 0.46 when wages are continuously renegotiated and 0.02 when wages are never renegotiated. Furthermore, when wages are infrequently renegotiated, much of the surplus captured by the worker comes from the response of turnover and not from their bargaining power. In the tail of the distribution, there are few firms paying the same wage, and there is only a small response of turnover to the wage. Hence, the labor share falls quickly in the tail of the distribution when wages are infrequently renegotiated. The implied productivity distribution will therefore have a much fatter right tail under infrequent renegotiation in order to match the same wage distribution.

In section 5, I endogenize the choice of the frequency of renegotiation by allowing each firm to optimally pick the contract length after meeting a worker. A higher contract length increases the wage that results from renegotiation. A firm can then influence the wage that results from a renegotiation by setting the frequency of renegotiation. The firm will then set a wage contract with infrequent renegotiation if the decrease in turnover due to a higher wage offsets the higher wage cost. The profits will then satisfy either (i) the envelop condition or (ii) the Nash bargaining solution with transferable values. The first case corresponds to infrequent renegotiation when firms have an incentive to pay a higher wage in all periods to reduce turnover. The second case of continuous renegotiation corresponds to the case in which firms would optimally set a lower wage in all periods and therefore set the most frequent renegotiation possible. The latter occurs if the bargaining power of the worker is high and the profits are small compared to the increased productivity from a better match. This is always the case for the worst match, who always renegotiates continuously. The infinite contract length, on the other hand, is optimal only if firms have all the bargaining power. Besides these cases, the optimal contract length is intermediate. When productivities are

period, the timing restriction means that this is an equilibrium result. Similarly, in Moscarini (2005) when a worker meets another firm the two firm engage in an auction for the worker. There is an arbitrary small cost to participate in the auction and in the equilibrium outcome the auction never occurs and the payoffs correspond to the Nash bargaining solution treating turnover as fixed.

homogeneous, then the extreme cases occur only if one of the agents has all the bargaining power.

The fact that the turnover, and hence the surplus, responds to the wage might imply that the bargaining set is non-convex. [Shimer \(2006\)](#) pointed out that since a convex bargaining set is one of the assumptions of [Nash \(1950\)](#), applying the Nash bargaining solution in models with OJS is problematic. [Shimer \(2006\)](#), instead, suggested a model with non-cooperative bargaining, as opposed to the usually applied cooperative Nash bargaining solution. There exists an equilibrium in his model, but it is not unique. Before [Shimer \(2006\)](#), the Nash bargaining solution was applied to models with OJS without accounting for the fact that the bargaining set might not be convex ([Pissarides, 1994](#); [Mortensen, 2003](#)). In the case of [Pissarides \(1994\)](#), the surplus was treated as fixed in the bargaining stage and therefore independent of the wage. In contrast, in the case of [Mortensen \(2003\)](#), the endogenous response of turnover, and thereby surplus, is taken into account. In the limit when there is no renegotiation, the equilibrium in my model corresponds to the equilibrium payoffs in [Mortensen \(2003\)](#) and one of the equilibria in [Shimer \(2006\)](#). If, instead, renegotiation becomes continuous and the length of the contract goes to zero, the equilibrium payoffs in my model correspond to the values in [Pissarides \(1994\)](#).

To see why the equilibrium is unique, it is useful to first analyze why the bargaining solution is unique given the equilibrium wage distribution, and then why the equilibrium wage distribution is unique. Given a wage distribution, a mapping between type and wage is a unique bargaining outcome if it is a fixed point in the following sense: given an expectation that the mapping between type and wage will hold in future wage renegotiations, the mapping between type and wage is the unique bargaining outcome in the current wage negotiation. In [Shimer's](#) model, there is an indeterminacy, as the absence of future wage renegotiation means that the future mapping between type and wage is payoff irrelevant in the current wage negotiation. However, with wage renegotiation, the expected outcome of future wage renegotiations becomes payoff relevant in the current bargaining stage, which means that firm type matters for the bargaining outcome. This distinction leads to a unique bargaining equilibrium given a wage distribution.

Turning to the uniqueness of the wage distribution, the solution to the bargaining game implies that the wage for each type correspond to the strict local maximum of the Nash product. This, in turn, implies that the function mapping a type to a wage can be characterized as a differential equation. To establish uniqueness, we need to establish that there is a unique initial condition. The initial condition for the lowest wage is given by the (unique) bargaining solution treating turnover as fixed. To show this, we proceed by contradiction. Assume that the lowest wage is lower than

the bargaining solution treating turnover as fixed. Note that, within the support of wages, the impact on firm profits from an increased wage is partly offset by endogenously lower turnover. This turnover effect increases the wage that is consistent with bargaining. A wage strictly lower than the bargaining solution treating turnover as fixed is therefore not a bargaining outcome. Similarly, assume that the lowest wage is higher than the bargaining solution treating turnover as fixed. To see why the lowest wage is not a bargaining outcome, note that below the support of wages, the Nash product is strictly concave and the maximum corresponds to the bargaining solution treating turnover as fixed. This implies that the Nash product increases for the lowest type as the wage is lowered at the lowest wage. The lowest wage is then not the strict local maximum of the Nash product and therefore, cannot be a bargaining outcome. Hence the lowest wage is given by the (unique) bargaining solution treating turnover as fixed.

In [Shimer](#)'s model, there is no unique minimum value of the wage distribution, as the bargaining outcome does not strictly maximize the Nash product locally. Indeed, the Nash product is constant across the whole wage distribution on the interior of the support. For a sufficiently small friction, an offer by the firm and the worker can be found on the interior on the support. This means that the argument above cannot be used to rule out a lowest wage that is higher than the wage treating turnover as fixed, and the wage distribution is then indeterminate.

In the literature, there are a number of alternative wage setting mechanisms in models with random OJS. The difference, in my model comes from the type of commitment available to the agents. In my model, the only commitment possible by the agents, is to the wage during the length of the contract. Wage posting models typically assume that firms have all the bargaining power, and that the wages are set only at the time of vacancy posting (i.e. no renegotiation). See, for instance, [Burdett and Mortensen \(1998\)](#).³ The wage is chosen so that the marginal gain from hiring and retaining workers exactly matches the increased wage cost. After hiring a worker, the firm has an incentive to change the agreed wage as it no longer affects the probability of hiring a worker. This means that wages are not time consistent. [Postel-Vinay and Robin \(2002\)](#) also consider a case in which the firm has all the bargaining power, but is able to observe outside offers and make counter-offers. When the worker receives an offer, the firm employing the worker and the other

³[Gautier et al. \(2010\)](#), like [Shimer \(2006\)](#), consider a model in which wages are set after the match is formed. The increased retention is then the only reason to increase pay. [Coles and Mortensen \(2016\)](#) consider a model in which hiring cost are independent of the wage the increased retention is then the only reason for increased pay irrespective of the timing of wage setting.

firm making the offer, engage in Bertrand competition over the worker. The equilibrium entails the worker moving to the most productive firm. The wage at this firm is such that the worker is indifferent to working at the less productive firm at a wage equal to its productivity. The wage thereby increases within a match, as counter-offers arrive. The model requires firms to commit to keeping the wage forever after the counter-offer has expired. This model is extended to include bargaining with a treat point equal to the value of the worst match in [Dey and Flinn \(2005\)](#) and [Cahuc et al. \(2006\)](#). In these models the wage does not affect the turnover of the worker. Values are therefore perfectly transferable. The lower productive firm has ex-post no incentive to participate in the competition for the worker as the worker will anyway move to the more productive only with a higher wage ([Moscarini, 2005](#)).

Section 2 defines the general model and expands on the contributions of the paper discussed above. Section 3 provides a closed form solution in the case of homogeneous productivities. Section 4 provides a quantitative evaluation of the model. Section 5 provides an extension of the model to a case in which the frequency of renegotiation is endogenous. Section 6 concludes.

2 Model

Environment. There is a frictional labor market with a continuum of two types of risk neutral and infinitely lived agents, firms and workers. Time is continuous and discounted at a rate ρ . A match between a worker and a firm differ in the type F , which I will also refer to as match quality. The type will determine the wage expectation and is observable by the agents. When a worker meets a firm, the type is drawn from the standard uniform distribution. A firm matched with a worker produces a flow output of $x(F)$, where the function $x(\cdot)$ is differentiable and weakly increasing. The flow profit is given by production $x(F)$ minus the agreed wage. Workers are homogeneous but differ in their employment state (unemployed or employed), the wage w , and the type F . An unemployed worker receives a flow benefit b and job offers at rate λ_u . An employed worker receives a wage w and job offers at rate λ_e . The job gets destroyed at rate δ in which case the worker becomes unemployed and the firm gets no payoff.

In contrast to [Burdett and Mortensen \(1998\)](#), [Mortensen \(2003\)](#), and [Shimer \(2006\)](#) I assume that a wage contracts do not last forever but are instead occasionally renegotiated. The wage in a match is renegotiated at a Poisson rate $\gamma(F)$ until which time it remains fixed, where $\gamma(F)$ is a weakly decreasing differentiable function (it will turn out, see section 5, that different types

optimally pick different contract lengths). At the time of renegotiation, a new wage is determined in a bargaining game with alternating offers between the worker and the firm. I restrict my attention to Markov strategies in the bargaining game and to equilibria in which the wage is weakly increasing in the match quality. $w(F)$ denotes the equilibrium bargaining outcome when the type is F . Lastly, I assume that if the worker is indifferent between the offer and her current job, she moves if the type is higher.

Value functions. Given an equilibrium wage function $w(\cdot)$, the value function for an employed worker, $W(F, w)$, is given by the expression

$$(\delta + \rho + \gamma(F))W(F, w) = w + \lambda_e \int_0^1 \max \left\{ W(\tilde{F}, w(\tilde{F})) - W(F, w), 0 \right\} d\tilde{F} + \delta U + \gamma(F)W(F, w(F)). \quad (1)$$

The value function of the worker thus depends on the flow income, the search option, and the value from renegotiation. Importantly, in the model, the worker observes the match quality F and forms rational beliefs about the wage in renegotiation. The value function for an unemployed worker, U , is given by

$$\rho U = b + \lambda_u \int_0^1 \max \left\{ W(\tilde{F}, w(\tilde{F})) - U, 0 \right\} d\tilde{F}, \quad (2)$$

where \tilde{F} denoted an arbitrary match quality from which an offer is received. The value function of the firm, $\Pi(F, w)$, is given by

$$(\delta + \rho + \gamma(F) + \lambda_e(1 - G(V(F, w))))\Pi(F, w) = x(F) - w + \gamma(F)\Pi(F, w(F)) \quad (3)$$

where $G(V) = \int_{\{\tilde{F}: V > V(\tilde{F}, w(\tilde{F})) \text{ or } V = V(\tilde{F}, w(\tilde{F})) \text{ and } \tilde{F} < F\}} d\tilde{F}$ is the fraction of match qualities with an implied value that is strictly less than $V(F, w)$ or where the value is the same and the type is less. Note that the firms expected profits are different even if the current wage is the same due to differences in (i) productivity, (ii) turnover, and (iii) the renegotiation value.

It is useful to work with the excess values of the match for the two types of agents. The excess value of the match to the firm is just the profits. The excess value to the worker, $V(F, w)$, is given by the value to the worker of the job, less the value of unemployment. That is,

$$V(F, w) = W(F, w) - U. \quad (4)$$

The normalized outside value for the worker, \bar{b} , is similarly defined as, the constant wage that makes a worker indifferent to unemployment. At such a wage, the difference in flow income exactly matches the difference in search option. \bar{b} is thus defined by the following equation

$$\bar{b} = b + (\lambda_u - \lambda_e) \int_0^1 \max \left\{ V(\tilde{F}, w(\tilde{F})), 0 \right\} d\tilde{F}. \quad (5)$$

The measure F is normalized to only include productivity levels that lead to matches. That is, the productivity satisfies $x(0) > \bar{b}$. The surplus, $S(F, w)$, of the match is the sum of the profits and the excess value to the worker. We can express the surplus as

$$\begin{aligned} (\delta + \rho)S(F, w) &= x(F) - \bar{b} - \lambda_e \int_0^1 \max \left\{ V(\tilde{F}, w(\tilde{F})), 0 \right\} d\tilde{F} + \gamma(F) (S(F, w(F)) - S(F, w)) \\ &+ \lambda_e \int_0^1 1 \left(V(\tilde{F}, w(\tilde{F})) \geq V(F, w) \right) \left(V(\tilde{F}, w(\tilde{F})) - V(F, w) - \Pi(F, w) \right) d\tilde{F}. \end{aligned} \quad (6)$$

The surplus of the match depends on the excess productivity, $x(F) - \bar{b}$, the renegotiation option, the search option as well, as the duration of the match. The last term in this expression reveals a bilateral inefficiency in the relation that is going to play a key role in the model. The worker moves whenever the value to the worker is higher at the new firm. But when the worker quits, the profits are lost to the pair. If profits are positive then the surplus of the match will be higher than the worker value. When $V(\tilde{F}, w(\tilde{F})) < S(F, w)$, the transition is bilaterally inefficient. Importantly, by agreeing on a higher wage, the worker will quit less often, and since these transitions are bilaterally inefficient the surplus of the match will be higher. Note that from the planner's perspective the quit rate is efficient as the worker keeps moving to more productive jobs.

Examining (6), reveals that the joint surplus is maximized if the worker moves if and only if the value to the worker is higher at the new job than the surplus in the current job. Depending on the type of commitment available to the agents a number of contracts can implement this. A simple contract that implements bilaterally efficient transitions entails the worker paying an upfront fee and subsequently being paid the productivity of the match, see also [Burdett and Coles \(2003\)](#) and [Stevens \(2004\)](#). In this case the profits of the firm is zero so the worker value is equal to the full surplus. Other, prominent examples are the models with sequential auctions, see for example [Postel-Vinay and Robin \(2002\)](#), in these models the outside option (bargaining position) of the worker is given by the full value of the previous match. The worker is thus able to extract the full profits of the current firm from the new firm. At the time of the transitions the profits associated with staying at the less productive job is zero and again the worker value captures the full value of the match.

If the wage function is strictly increasing in match quality we can express the marginal increase in surplus with the match quality as

$$\frac{\partial S(F, w(F))}{\partial F} = \frac{x'(F) + \lambda_e \Pi(F, w(F))}{\delta + \rho + \lambda_e(1 - F)}. \quad (7)$$

The first term captures the increase in the surplus coming from increased productivity. The second

term captures the increase in surplus due to fact that profits are retained when the worker quits less often, this term will be higher if the job offer arrival rate or the level of profits of the firm are high.

Bargaining game. Bargaining occurs in discrete artificial time as in [Shimer \(2006\)](#). This will ensure that the model payoffs correspond to the standard Nash bargaining solution in a stochastic environment without OJS, see [Coles and Muthoo \(2003\)](#). The set-up closely follows [Binmore et al. \(1986\)](#). The players alternate in making offers. After the proposer has made an offer, the responder chooses to accept or reject the offer. If the offer is accepted, the agents get the payoffs associated with the agreed wage. If the offer is rejected, we move to the next bargaining period and there is a probability that the bargaining process breaks down. If the process breaks down, the parties get their outside option. The probability that there is no breakdown is $(1 - \Delta)^\beta$ after the worker makes an offer and $(1 - \Delta)^{1-\beta}$ after an offer by the firm. β determines the relative bargaining power of the worker.

During the bargaining process, the firm and the worker take the outcome of future wage negotiations as fixed, which means that the functions V , U and Π are to the firm and worker fixed. The bargaining game consists of two players: a firm with payoff function Π and a worker with (excess) payoff function V . The action set is \mathbb{R}_+ for the proposer and $\{\text{Accept, Reject}\}$ for the responder. I will examine sub-game perfect equilibrium (SPE) in Markov strategies. A Markov strategy is such that the offer and acceptance rules only depend on the match quality and not on the previous history. I define $w_\Delta(F)$ to be a bargaining outcome associated with a Markov SPE when the friction is Δ . Definition 1 defines the limit outcome of the bargaining game.

Definition 1 *$w(F)$ is a limit outcome of the bargaining game if, for all $\epsilon > 0$, there exists a $\bar{\Delta}$ such that $|w(F) - w_\Delta(F)| < \epsilon$ for all $\Delta < \bar{\Delta}$.*

Let $w_{i,\Delta}(F)$ denote the wage offer by agent i in the bargaining game with friction Δ and match quality of F , for $i \in \{w, f\}$, where w and f refers to the workers and firms, respectively.

Equilibrium. For an equilibrium I require that $w(\cdot)$ is an outcome of a Markov SPE in the bargaining game as the friction goes to zero.

Definition 2 *The wage function $w(F)$ is equilibrium if for all $F \in [0, 1]$ $w(F)$ is the limit outcome of the bargaining game.*

In an equilibrium, the firm and the worker make offers such that the value function of the responder evaluated at the offer is equal to their continuation value. A wage offer associated with

a value less than the continuation value is not accepted, and, given the costly delay, such an offer is not optimal. Similarly, an offer higher than the continuation value is accepted, but results in a smaller payoff for the proposer. The following theorem summarizes these results.

Theorem 1 *There exists a unique equilibrium which satisfies the differential equation*

$$\beta \Pi(F, w(F)) \frac{\partial V(F, w)}{\partial w} \Big|_{w=w(F)} + (1 - \beta) V(F, w(F)) \frac{\partial \Pi(F, w)}{\partial w} \Big|_{w=w(F)} = 0, \quad (8)$$

with the initial condition

$$\beta \Pi(0, w(0)) = (1 - \beta) V(0, w(0)). \quad (9)$$

For all $F \in [0, 1]$ and a sufficiently small Δ , the two offers solve

$$V(F, w_{f,\Delta}(F)) = (1 - \Delta)^{(1-\beta)} V(F, w_{w,\Delta}(F)) \quad (10)$$

$$\Pi(F, w_{w,\Delta}(F)) = (1 - \Delta)^\beta \Pi(F, w_{f,\Delta}(F)), \quad (11)$$

with $w_{f,\Delta}(F) < w(F) < w_{w,\Delta}(F)$ and $\lim_{\Delta \rightarrow 0} w_{f,\Delta}(F) = \lim_{\Delta \rightarrow 0} w_{w,\Delta}(F) = w(F)$.

Discussion of uniqueness. The formal proof of the Theorem 1 is presented in Appendix A.1. The bargaining outcome for a given match is unique. In order to see why, it is useful to combine (10) and (11) to obtain

$$V(F, w_{f,\Delta}(F))^\beta \Pi(F, w_{f,\Delta}(F))^{1-\beta} = V(F, w_{w,\Delta}(F))^\beta \Pi(F, w_{w,\Delta}(F))^{1-\beta}. \quad (12)$$

We see that the Nash product is the same when evaluated at either the firm's or the worker's offer. Furthermore, taking the limit as the friction goes to zero we get (8). At the bargaining outcome, the derivative of the Nash product with respect to the wage is zero. It is useful to define $w(F, V)$ as the wage that gives a worker in a match with type F a value V . For this wage to be a bargaining outcome we require that the derivative of the Nash product is zero. Substituting for the derivative of the value functions we can rewrite the necessary condition as

$$\frac{\beta}{V} - \frac{(1 - \beta)}{\Pi(F, w(F, V))} - \frac{(1 - \beta) \lambda_e G'(V)}{\delta + \rho + \gamma(F) + \lambda_e (1 - G(V))} = 0. \quad (13)$$

Assume that V is the value associated with the bargaining outcome for a match $F' < F$. Worker turnover is less for a match type F after renegotiation and the productivity is weakly higher. The profits are therefore higher for F than for F' when evaluated at the same worker value, $\Pi(F, w(F, V)) > \Pi(F', w(F', V))$. Similarly, the contracts last (weakly) longer for F . Combining both of these insights implies that if the derivative of the Nash product with respect to the wage is

zero for F' , it cannot be zero for F (this is discussed more formally in Appendix A.1). In particular the derivative is positive if $F > F'$ and negative if $F < F'$.

Thus, we see that the inclusion of renegotiation in the model of Shimer (2006) implies that the bargaining outcome within a match is unique. This occurs because the wage outcome in future renegotiations affects the value functions today and in particular, in equilibrium, the future expectations will uniquely pin down the bargaining outcome. In contrast, in Shimer's model, there is no renegotiation, so the value functions are independent of the type when productivities are the same, i.e. $\Pi(F, w(F, V)) = \Pi(F', w(F', V))$.

The unique equilibrium in this model is such that the wage function solves the differential equation in Theorem 1, where the initial condition for the differential equation is given by the bargaining outcome that arises if turnover is treated as fixed. To see why there is a unique equilibrium, it is useful to consider the bargaining game with the lowest match quality. The argument in the previous paragraph implies that, on the interior of the support, the Nash product has a strict local maximum at the bargaining outcome. Hence, for the lowest type firm, the Nash product decreases as the wage increases above the lowest wage. The bargaining game implies that the Nash product must be the same at the offer by the worker and the firm. The Nash product must therefore be increasing in the wage below the lowest wage. Outside the support the maximum occurs at (9) and the lowest wage must therefore be weakly lower than this value. The bargaining outcome with OJS, results in a wage that is greater or equal to the bargaining outcome treating turnover as fixed (for the lowest type this implies a wage weakly greater than that which solves (9)). The unique initial condition must therefore solve (9).

It is now useful to discuss why the inclusion of renegotiation results in a unique equilibrium wage distribution even if all jobs have the same productivity. As discussed in the previous paragraph, in Shimer (2006), the firms are not distinct and the Nash product is constant on the support of wages. If the Nash product is constant on the support of wages then for an (sufficiently) small probability of breakdown, an offer by the worker and firm, on the support of wages, can then be found such that (10) and (11) hold. Letting the friction go to zero the offers can then converge to an arbitrary point on the distribution, unlike in my model where they converge to a unique point. In particular, the two offers can converge to the initial condition. Shimer's model therefore gives a differential equation, implying constant Nash product, but not an initial condition. This is similar to the wage posting model where the firm's optimal choice of the wage implies a differential equation for the wage function. In order to get the initial condition for the wage function we consider a deviation

below the support of the wage distribution. Such an offer must not be accepted by the worker and it must therefore be that the worker is indifferent between the lowest wage and unemployment. In the present model, the fact that all match qualities are distinct implies that the offer by the firm falls outside the support of wages and there is a unique equilibrium where the bargaining outcomes corresponds to the global maximum of the Nash product. [Shimer](#) conjectured that, since there is a unique initial condition for his differential equation such that the Nash product is a local maximum for all wages, the limit of a model with heterogeneous productivities as firms become homogeneous, might results in a unique equilibrium. In this paper, I show that introducing an arbitrary small probability of renegotiation results in the conjectured equilibrium in [Shimer](#) (the proof of this goes through if productivities are heterogeneous but there is no renegotiation, thereby verifying [Shimer's](#) conjecture).

One might worry about a discontinuity in the number of equilibria in the limit. There are alternative refinements to the model in [Shimer \(2006\)](#) that result in an unique equilibrium. Trivially, by examining equilibria where, in lowest type match, the offer by the firm is below the support of wages, results in the same unique equilibrium. Alternatively, considering equilibria that are the limit from an arbitrary large initial friction also results in the same unique equilibrium. With a sufficiently large (initial) friction the two offers converge to the global maximum of the Nash product. This corresponds to the bargaining outcome only if the initial condition is given by (9).

Discussion of the equilibrium. We can now turn to analyzing the properties of the solution. Using the derivative of the value functions we can rewrite the bargaining equation, using the surplus $S(F, w(F)) = \Pi(F, w(F)) + V(F, w(F))$, as⁴

$$\Pi(F, w(F)) = \frac{(1 - \beta) \left[1 - \frac{\lambda_e}{w'(F)} \frac{\delta + \rho + \lambda_e(1-F)}{\delta + \rho + \gamma(F) + \lambda_e(1-F)} \Pi(F, w(F)) \right]}{\beta + (1 - \beta) \left[1 - \frac{\lambda_e}{w'(F)} \frac{\delta + \rho + \lambda_e(1-F)}{\delta + \rho + \gamma(F) + \lambda_e(1-F)} \Pi(F, w(F)) \right]} S(F, w(F)) \quad (14)$$

$$V(F, w(F)) = \frac{\beta}{\beta + (1 - \beta) \left[1 - \frac{\lambda_e}{w'(F)} \frac{\delta + \rho + \lambda_e(1-F)}{\delta + \rho + \gamma(F) + \lambda_e(1-F)} \Pi(F, w(F)) \right]} S(F, w(F)). \quad (15)$$

Compared to bargaining without OJS, there is an extra term coming from the fact that the wage affects turnover. This terms is given by $\frac{\lambda_e}{w'(F)} \frac{\delta + \rho + \lambda_e(1-F)}{\delta + \rho + \gamma(F) + \lambda_e(1-F)} \Pi(F, w(F))$ and results in the worker receiving a higher share of the surplus. This term captures the increase in surplus with a higher wage. If the worker leaves for a marginally better firm then the profits are lost. This marginal increase in turnover is therefore not bilaterally efficient. A small reduction in turnover increases the

⁴Where we use that $G'(V(F, w(F))) = (\delta + \rho + \lambda_e(1 - F))/w'(F)$ and $\frac{\partial V(F, w)}{\partial w}|_{w=w(F)} = (\delta + \rho + \gamma(F) + \lambda_e(1 - F))^{-1}$

match surplus by the change in turnover multiplied by the level of profits. The change in turnover is given by the density of incoming wage offers $\lambda_e/w'(F)$ multiplied by the fraction of the duration of the match that the wage remains fixed for $\frac{(\delta+\rho+\lambda_e(1-F))}{\delta+\rho+\gamma(F)+\lambda_e(1-F)}$, reflecting how the worker trades off a higher wage against the type. As the length of the contract decreases, the wage becomes less important, compared to the match quality, for the worker. As the length of the contract decreases, turnover becomes less responsive to the wage and workers capture a smaller share of the surplus.

In the limit, as the contract length goes to zero ($\gamma(F) \rightarrow \infty$), the surplus becomes unresponsive, the wage then solves the standard Nash bargaining solution with perfectly transferable utilities, given by (16) and (17) below

$$\Pi(F, w(F)) = (1 - \beta)S(F, w(F)) \quad (16)$$

$$V(F, w(F)) = \beta S(F, w(F)). \quad (17)$$

The model thus provides a justification, based on continuous renegotiation, for using the Nash bargaining solution with perfectly transferable values, as in [Pissarides \(1994\)](#). Intuitively, it is only future wages that reduce turnover and as renegotiation becomes very frequent, the current wage becomes irrelevant for future wages. Similarly as the length of the contract goes to infinity ($\gamma(F) \rightarrow 0$) and bargaining powers is made symmetric, the differential equation corresponds to that in [Mortensen \(2003\)](#) and [Shimer \(2006\)](#). Since wages are never renegotiated, the worker only cares about the wage and not about the match quality. Importantly, if wages are only sometimes renegotiated then the solution corresponds to neither [Pissarides \(1994\)](#) nor [Shimer \(2006\)](#).

3 Homogeneous productivities

In this section, I impose some restrictions on the general model. Firstly, I assume homogeneous productivity of firms (i.e. $x(F) = x > \bar{b}$). Secondly, the discounted duration of a wage contract is a fixed fraction, θ , of the expected discounted duration of the job. This is a useful benchmark for varying the amount of commitment available to the agents as all types are influenced symmetrically. That is, under this specification a small increase in the wage by $w'(F)dw$, decreases turnover by θdw . θ thus captures the (marginal) relative importance of the wage and the type. If the contract length, $\gamma(F)$, was constant across types, then the (marginal) relative importance of the type compared to the wage would be higher for higher types. When θ is one, the model corresponds to the case of no renegotiation, as in [Shimer \(2006\)](#), and the limit as θ goes to zero corresponds to continuous

renegotiation. The arrival rate of renegotiation then solves

$$\gamma(F) = \frac{1-\theta}{\theta}(\delta + \rho + \lambda_e(1-F)).$$

For this specific model, we have a closed-form analytical solution. Theorem 2 presents the distribution function, the inverse of the wage function, and the value functions.⁵

Theorem 2 *The wage offer distribution is given by*

$$F(w) = \frac{(\delta + \rho + \lambda_e)}{\lambda_e} \left(1 - \left[\frac{x-w}{x-\underline{w}} \right]^{1/\theta} \left[1 - \frac{(x-\underline{w})^{1/\theta}}{\underline{w}-\bar{b}} \left(\frac{1-\beta+\beta/\theta}{1-\beta} \right) \frac{(x-w)^{1-1/\theta} - (x-w)^{1-1/\theta}}{1-1/\theta} \right]^{\frac{\beta}{\theta(1-\beta)+\beta}} \right)$$

and the value functions are given by

$$\begin{aligned} \Pi(F(w), w) &= \frac{x-w}{(\delta + \rho + \lambda_e) \left[\frac{x-w}{x-\underline{w}} \right]^{1/\theta} \left[1 - \frac{(x-\underline{w})^{1/\theta}}{\underline{w}-\bar{b}} \frac{1-\beta(1-1/\theta)}{(1-\beta)} \frac{(x-w)^{1-1/\theta} - (x-w)^{1-1/\theta}}{(1-1/\theta)} \right]^{\frac{\beta}{\theta(1-\beta)+\beta}}} \\ V(F(w), w) &= (\underline{w} - \bar{b}) \frac{\left[1 - \frac{(x-\underline{w})^{1/\theta}}{\underline{w}-\bar{b}} \left(\frac{1-\beta+\beta/\theta}{1-\beta} \right) \frac{(x-w)^{1-1/\theta} - (x-w)^{1-1/\theta}}{1-1/\theta} \right]^{1-\frac{\beta}{\theta(1-\beta)+\beta}}}{\delta + \rho + \lambda_e}, \end{aligned}$$

where $\underline{w} = \beta(x - \bar{b}) + \bar{b}$.

Theorem 2 extends the solution in Shimer (2006) to incorporate renegotiation and different bargaining powers. In order to illustrate how the model behaves, I pick parameter values that are typically used in the literature to model the US economy. The job arrival rate and the job destruction rate are picked to match the job finding rate and the unemployment rate. Similarly, the job offer arrival rate in employment is picked to match the rate at which workers change employer. I pick an annual discount rate of 5%.⁶ A crucial parameter in the model is the job offer arrival rate in employment over the separation rate, which measures the effective amount of competition for workers. I pick the bargaining power, β such that the worker captures 2/3 of the difference between x and \bar{b} when θ is equal to a half. Furthermore, x and \bar{b} are normalized to

⁵The solution comes from solving (8) with the initial condition (9) where I use that

$$\begin{aligned} V_w(F, w)|_{w=w(F)} &= 1/(\delta + \rho + \gamma(F) + \lambda_e(1-F)) \\ (\delta + \rho + \gamma(F) + \lambda_e(1-G(V(F, w(F))))\Pi_w(F, w) &= -1 + \lambda_e G_V(V(F, w))V_w(F, w)\Pi(F, w) \\ G_V(V(F, w(F)))V_w(F, w)|_{w=w(F)} &= \frac{\partial V(F, w)}{\partial w}|_{w=w(F)} \left[\frac{\partial V(F, w(F))}{\partial F} \right]^{-1} = \frac{1}{w'(F)}\theta \end{aligned}$$

⁶Note that the discount rate and the separation rate enter the worker and firm problem in the same way but the separation rate is an order of magnitude more important. The discount rate has therefore of little quantitative importance.

Table 1: Parameters and moments

	Value	Moment	Reference
λ_u	0.45	45% Monthly job finding rate	Shimer (2012)
λ	0.181	3.2% Monthly job-to-job transition rate	Moscarini and Thomsson (2007)
δ	0.024	5% unemployment rate	—
ρ	0.004	5% annual discount rate	—
β	0.161	2/3 labor share	—

Source: The job finding rate refers to the average over the period 1948Q1-2007Q1, as calculated by [Shimer \(2012\)](#).

one and zero, respectively. The parameter values are presented in Table 1. In Figure 1, I plot the wage function and the worker’s and firm’s value function for different values of the frequency of renegotiation holding the other parameter values, including the bargaining position (\bar{b}) of the worker, fixed.

When renegotiation is continuous, and productivities are homogeneous, then the surplus still increases with the type due to the lower turnover. This results in a increasing wage function, unlike the case of no OJS when the function would be flat. The more infrequent the renegotiation, the steeper is the wage function. The worker’s value function exhibits the same behavior. Worker turnover does not, in equilibrium, depend on the frequency of renegotiation but wages decrease if renegotiation becomes more frequent. The profits are increasing in the frequency of renegotiation and particularly so for higher types (there is in fact no change for the lowest type). In particular, for frequent renegotiation, profits increase in type and if renegotiation becomes sufficiently infrequent, profits decrease with type. Similarly, if renegotiation becomes more frequent, the Nash product increases more with the type. In the case of no renegotiation, the Nash product is constant ([Shimer, 2006](#)). A central question must then be to find an empirically relevant range for the frequency of renegotiation.

4 Calibration

The previous sections analyzed the how the wage distribution and the firm’s and worker’s value depend on the frequency of renegotiation. In this section I aim to quantitatively assess the how the estimated primitives of the model depend on the assumed frequency of renegotiation. The key

Figure 1: Functions for homogeneous productivities

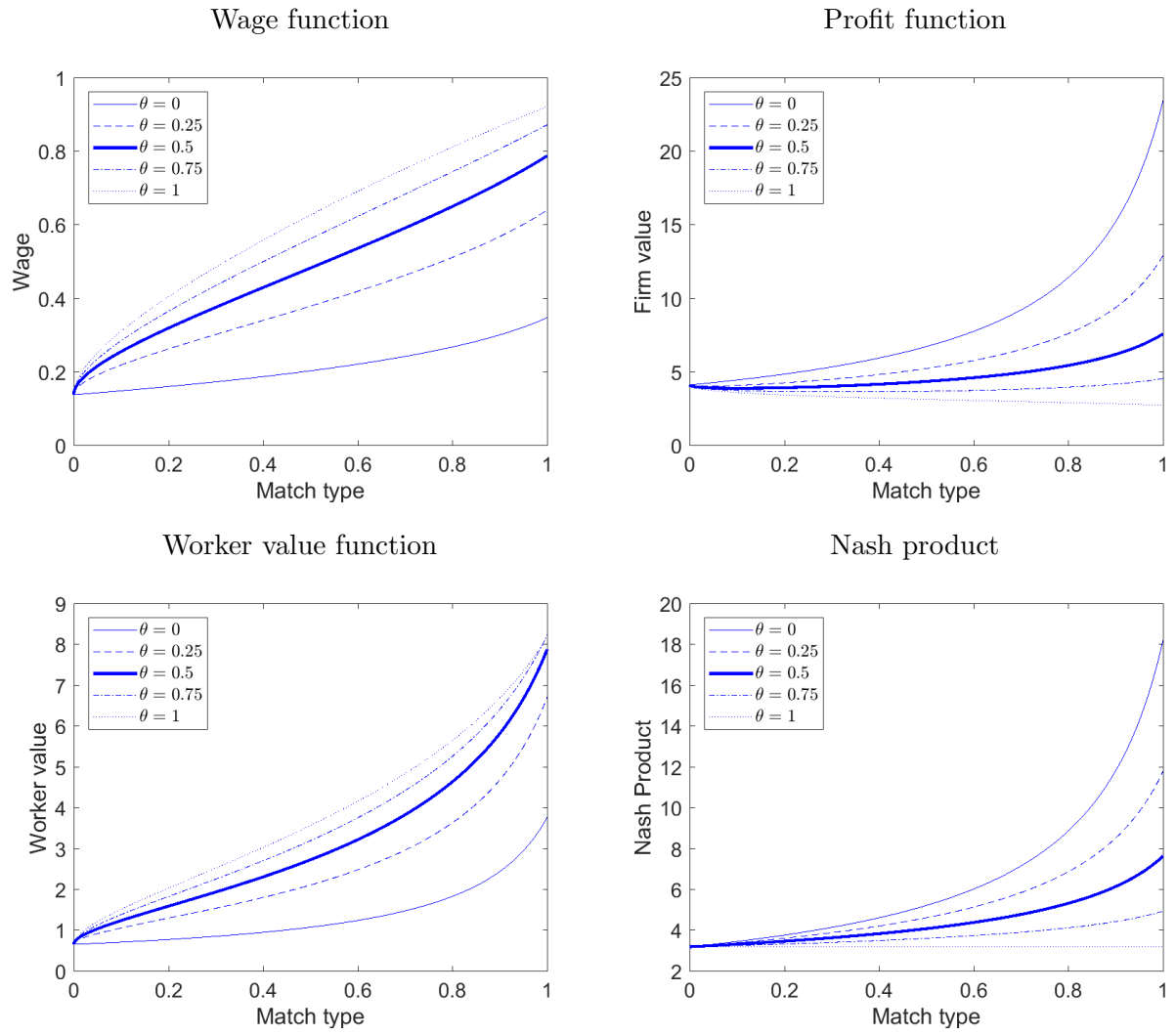


Table 2: Wage distribution

	Normal	Gumbel	Moment	Reference
σ	0.16	0.105	scale parameter	Gottfries and Teulings (2017)
\bar{F}	0.019	$1.3e - 06$	1.7 mean-min	Hornstein et al. (2007)

parameters of interest will be the bargaining power of the worker and the productivity distribution. I use the same transition parameters as in the previous section. I take the estimate of the scale parameter of the wage offer distribution from [Gottfries and Teulings \(2017\)](#), assuming that log wages are Normally and Gumbel distributed, which they find to provide a good fit for the data. Their method predominantly uses information from the the upper tail of the wage distribution and is therefore less suitable for assessing the lowest wage. This is instead the focus of [Hornstein et al. \(2007\)](#) who find that the mean wage is about 1.5 to 2 times larger than the lowest wage. I use a truncation parameter to target a mean-min ratio of 1.7. The targeted wage offer, $\hat{w}(F)$, is then given by

$$\hat{w}(F) = \Gamma^{-1}(F(1 - \bar{F}) + \bar{F}; \sigma) \quad (18)$$

where Γ refers to the inverse of the CDF (Normal or Gumbel), \bar{F} and σ denoted the truncation and scale parameter, respectively. Table 2 presents the parameters of the wage distribution.

Using only wage data it is not possible to identify the bargaining power separately from the productivity distribution. As in the previous section, I target a labor share of 2/3 to determine the bargaining power of the worker. Note that in so far as there is capital that is not specific to the worker (and therefore not lost in case of a bargaining breakdown) the bargaining power of the worker would have to be higher to match the same income share. We therefore have a conservatively estimated bargaining power of the worker. I target the 1-99th percentile in the log wage distribution with equal weights. I use a beta distribution for the log productivity with a scale and location parameter. That is the productivity of a match quality F satisfies

$$x(F) = \exp[\mu_x + \sigma_x B^{-1}(F; \alpha_x, \beta_x)] \quad (19)$$

where $B^{-1}(F; \alpha_x, \beta_x)$ is the inverse of the beta distribution with parameters α_x and β_x . The transition parameters are the same as in the previous section. In the estimation, the bargaining position \bar{b} is set to match the lowest wage exactly. I estimate, the model assuming that wages are (i) continuously, (ii) annually, and (iii) never renegotiated.

Table 3: Parameters

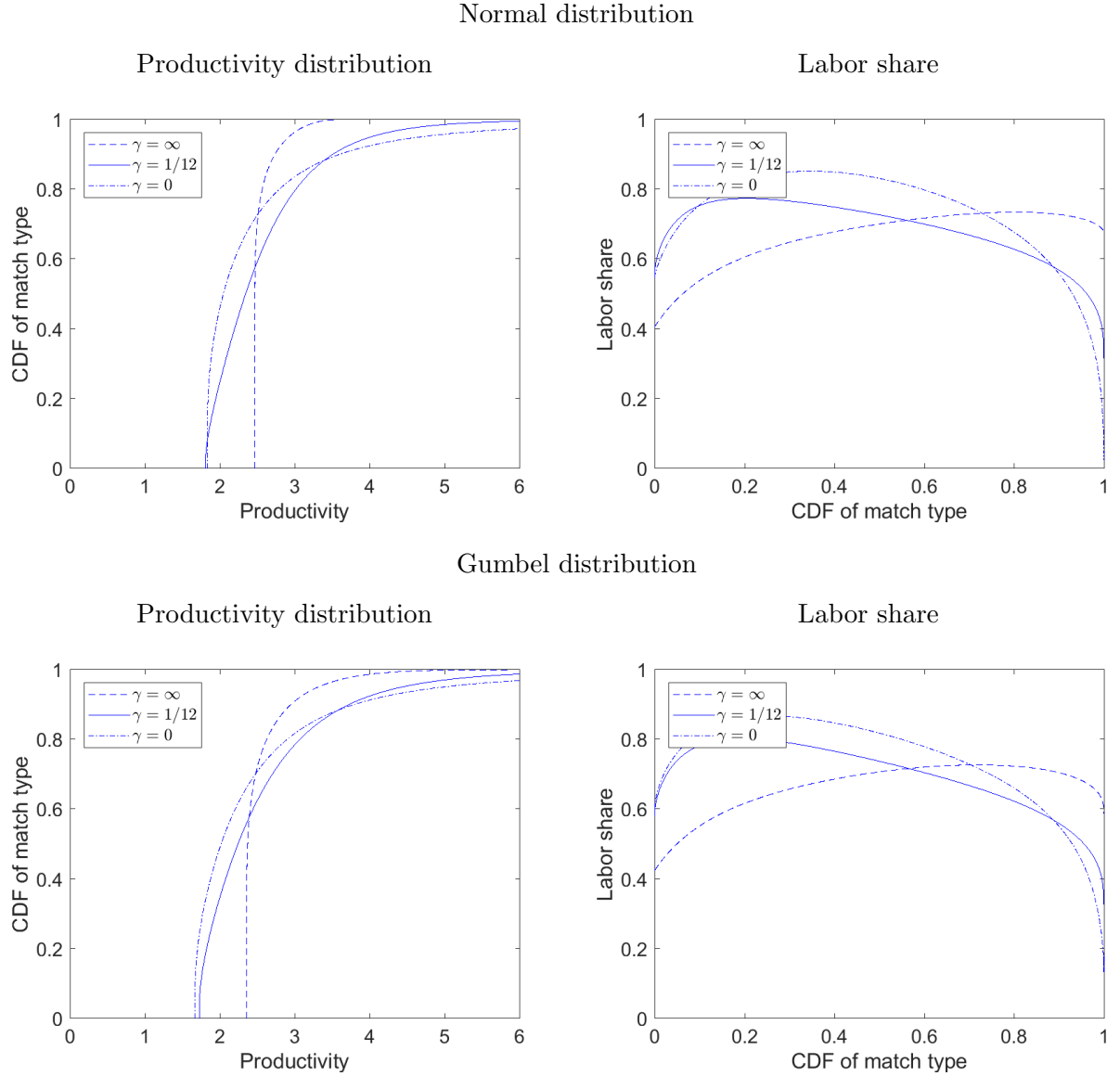
	Normal			Gumbel		
	$\gamma = \infty$	$\gamma = 1/12$	$\gamma = 0$	$\gamma = \infty$	$\gamma = 1/12$	$\gamma = 0$
α_x	0.02	0.3	0.09	0.03	0.22	0.14
β_x	1.69	7.8	7002004.68	2.74	6.65	4602719.78
μ_x	0.9	0.59	0.61	0.86	0.55	0.51
σ_x	0.36	2.36	3471404.49	0.91	2.69	2234479.69
β	0.46	0.19	0.02	0.45	0.23	0.12
\bar{b}	-0.24	0.81	0.98	-0.13	0.78	0.91

Note: The tables shows the productivity parameters and bargaining power of the worker. γ refers to the assumed Poisson rate by which wages are renegotiated and the value of the transition parameters are presented in Table 2.

Table 3 presents the estimated parameters. The estimated bargaining power depends crucially on the assumed frequency of renegotiation. Unless there is no renegotiation and wage offers are log Normally distributed, the bargaining power of worker is positive in order for the model to match a labor share of 2/3. Given the prevalence of wage posting models, where firms have all the bargaining power and the retention and hiring motives are the only reason to increase pay, this needs to be qualified. The longer the wage contracts last for, the lower is the estimated bargaining power of the worker. This comes from the fact that with infrequently renegotiated wages workers capture a higher share surplus due to the endogenous response of turnover.

The differences in bargaining power has important implications for the estimated primitives of the model. Figure 2 shows the implied firm productivity distribution for the different frequency of renegotiation and also the labor share at different firms. When the bargaining power of the worker is low and wages are very infrequently renegotiated then much of the wage is coming from the endogenous response of turnover to the wage. In the tail of the distribution there are few firms paying the same wage and hence turnover responds very little to the wage. This implies that the worker captures a very small share of the surplus. Indeed, if the density goes to zero then the labor share goes to the bargaining power. This is illustrated in the right panels which shows that the labor share falls much more in the tail of the wage distribution when wages are infrequently renegotiated. In order for the model with infrequently renegotiated wages to match the same wage distribution

Figure 2: Productivity distribution and Labor share

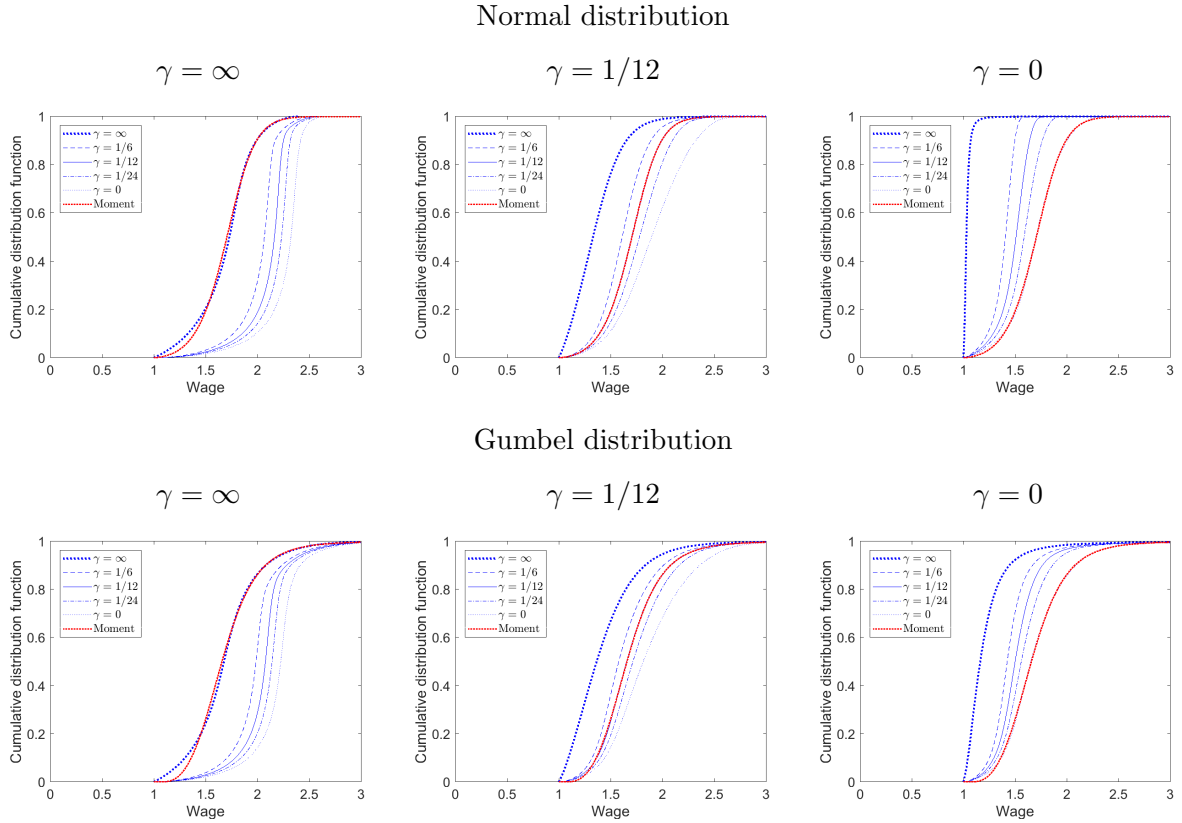


Note: The left panels shows the steady state CDF of the productivity distribution of outstanding matches. The right panel shows the labor share as a function of the rank of match quality in the steady state distribution. γ refers to the Poisson rate by which wages are renegotiated and the value of the other parameters are presented in Table 2 and 3.

it must then be that the productivity distribution has a fatter right tail. Indeed, [Mortensen \(2003\)](#) pointed out that the labor share is too small in the tail of the distribution in an estimated wage posting model.

I now illustrate the response of the wage distribution to a change in the contract length. Figure

Figure 3: Wage distributions



Note: The title refers to the contract length that is assumed when the model is estimated. Moment shows the targeted wage distribution. The figures show the counterfactual wage distribution that is implied by a change in the contract length holding all parameters fixed. γ refers to the Poisson rate by which wages are renegotiated and the value of the other parameters are presented in Table 2 and 3.

3 provides the wage distribution counterfactual wage distribution if the contract lengths is changed, holding the parameters values, as well as the bargaining position of the worker, fixed. This is a useful comparison as the government has separate instrument for these, e.g. by adjusting the benefit level and subsidizing hiring. The wage distribution with infrequently renegotiated wages, stochastically dominates the wage distribution when there is more frequent renegotiation. The results suggest that a change to the contract length can have a large impact on the wage distribution and particularly so if wages were initially never renegotiated.

5 Endogenous contract length

In this section I extend the model by allowing the firm to optimally pick the length of the contract after meeting the worker and having seen the type of the match. After the firm has decided on the contract length, that will last throughout the match the firm and the worker bargain over a wage. When deciding on the length of the contract, the firm takes the wage function at other firms as given. The firm, on the other hand, internalizes that the bargaining outcome will change with the contract length. Note that, unlike the model in the previous section, the firm can affect the wage outcome in future renegotiations by changing the contract length. At the margin an increase in the contract length will increase the wage. The firm may prefer a higher wage than the wage resulting from continuous renegotiation in order to reduce turnover. Then the contract length will be positive and the profits will satisfy the standard envelop condition. In effect the firm is setting the wage to maximize profits. Alternatively, it could be that the firms would optimally set a lower wage than the wage resulting from continuous renegotiation. In this case the wages will be continuously renegotiated and solve the Nash bargaining solution with transferable values. In the solution, profits either satisfy: (i) the standard envelop condition or (ii) the Nash bargaining solution with perfectly transferable values:

$$\frac{\partial \Pi(F, \gamma(F), w(F))}{\partial F} = \min \left\{ \frac{x'(F)}{\delta + \rho + \lambda_e(1 - F)}, (1 - \beta) \frac{x'(F) + \lambda_e \Pi(F, \gamma(F), w(F))}{\delta + \rho + \lambda_e(1 - F)} \right\}. \quad (20)$$

Integrating (20) together with the initial condition (21)

$$\Pi(0, \gamma(0), w(0)) = (1 - \beta) \frac{x(0) - \bar{b}}{\delta + \rho + \lambda_e} \quad (21)$$

gives the level of profits. Having solved for the level of profits we can integrate (7) to get a simple expression for surplus

$$S(F) = \int_0^F \frac{x'(\tilde{F}) + \lambda_e \Pi(\tilde{F}, \gamma(\tilde{F}), w(\tilde{F}))}{\delta + \rho + \lambda_e(1 - \tilde{F})} d\tilde{F} + \frac{x(0) - \bar{b}}{\delta + \rho + \lambda_e}. \quad (22)$$

Using the expression for the bargaining solution, (14), we then get an expression for the contract length that would rationalize $\Pi(F, \gamma(F), w(F))$ and $S(F, \gamma(F), w(F))$. Theorem 3 states this result and the proof is presented in Appendix A.2.

Theorem 3 *There exist an equilibrium satisfying (20), (21), and (22). In the equilibrium the contract length is given by*

$$\gamma(F) = (\delta + \rho + \lambda_e(1 - F)) \frac{\beta \Pi(F, \gamma(F), w(F))}{(1 - \beta) V(F, \gamma(F), w(F)) - \beta \Pi(F, \gamma(F), w(F))}. \quad (23)$$

A few aspects are interesting to note. Firstly, the lowest match quality always continuously renegotiates. If the productivities increases quickly with match quality compared to the level of profits or workers have a high bargaining power, then firms will opt for continuous renegotiation. If workers have all the bargaining power then this will always be the case. When firms have all the bargaining power, the case analyzed in [Gautier et al. \(2010\)](#), the optimal contract from firm's perspective is to never renegotiate the wage. Except in these cases, there will be an intermediate value for the frequency of renegotiation. Lastly, conditional on the asset values the optimal contract length increases in the bargaining power of the firm.

I now illustrate this for case of homogeneous productivities. For this case the solution is particularly easy, as all contract lengths will imply the same profits. The profits of the lowest type firm equals

$$\Pi(0) = (1 - \beta) \frac{x - \bar{b}}{\delta + \rho + \lambda_e}. \quad (24)$$

Since this must also be the profits for the other match types we get the simple expression for the surplus

$$S(F) = \frac{x - \bar{b}}{\delta + \rho + \lambda_e} \left(1 + (1 - \beta) \ln \left(\frac{\delta + \rho + \lambda_e}{\delta + \rho + \lambda_e(1 - F)} \right) \right). \quad (25)$$

Using the expression for the bargaining solution, (14), we get that the contract length must satisfy

$$\gamma(F) = \frac{\beta}{1 - \beta} \frac{(\delta + \rho + \lambda_e(1 - F))}{\ln \left(\frac{\delta + \rho + \lambda_e}{\delta + \rho + \lambda_e(1 - F)} \right)} \quad (26)$$

For the case of homogeneous productivities there is always in intermediate region for the frequency of renegotiation unless one of the parties has all the bargaining power. The optimal contract length increases in the bargaining power of the firm and also in the match quality.

6 Conclusion

In this paper, I generalize [Shimer's \(2006\)](#) model with OJS and bargaining to include renegotiation and different bargaining powers. In doing so, I break the indeterminacy found by [Shimer](#). In the model, a higher wage results in lower turnover and this results in the worker capturing a higher share of the surplus. The increase in wages depends on the fraction of the duration of the match for which the wage contract last. In the previous literature, either it is assumed that the fraction is zero or one. I estimate the model to show that the primitives of the model differ

starkly depending on the assumed contract length. Furthermore, if firms could optimally pick the frequency of renegotiation, the equilibrium would, in general, not be at the two extremes analyzed in the literature (no renegotiation and continuous renegotiation). Lastly, the paper provides a justification for using the Nash bargaining solution with transferable utilities in models with OJS, as that corresponds to the special cases of the model when wages are continuously renegotiated.

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A Proofs

A.1 Theorem 1

I prove Theorem 1 using a number of steps. Before moving to the proof, it is helpful to establish some notation. Define the equilibrium value function as

$$\bar{V}(F) = V(F, w(F)) \quad \text{and} \quad \bar{\Pi}(F) = \Pi(F, w(F)).$$

It is useful to define the wage $w(F, V)$ which gives the worker excess value V when in a match with quality F . The derivative of the workers value function with respect to the wage when in a match with quality F is strictly positive and equals

$$\left. \frac{\partial V(F, w)}{\partial w} \right|_{w=w(F, V)} = \frac{1}{\delta + \rho + \gamma(F) + \lambda_e(1 - G(V))}.$$

For a match of quality F it is useful to denote $\bar{x}(F)$ the wage that gives the firm zero profits and $\bar{b}(F)$ the wage that makes the worker indifferent between unemployment.

This proof builds on a number of propositions. The first proposition shows that the differential equation

$$\beta \Pi(F, w(F)) \left. \frac{\partial V(F, w)}{\partial w} \right|_{w=w(F)} + (1 - \beta) V(F, w(F)) \left. \frac{\partial \Pi(F, w)}{\partial w} \right|_{w=w(F)} = 0, \quad (27)$$

with the initial condition

$$\beta \Pi(0, w(0)) = (1 - \beta) V(0, w(0)) \quad (28)$$

constitutes a solution to an equilibrium. The second proposition shows that any equilibrium must satisfy the differential equation. In the third proposition I show that the initial condition must also hold. Lastly I show that for each match quality F there is a unique bargaining outcome when the firm and the worker, in the bargaining stage, take the wage distribution as given.

Proposition 1 *Suppose that a wage function satisfies (27) and (28) then an equilibrium exist.*

Proof. The proof is presented using a number of Lemmas.

Lemma 1 *If the wage function is strictly increasing in match quality then for any pair of types $F' < F$, the profit function is higher for the higher match quality when the profit functions are evaluated at the same worker value, when the worker value is an equilibrium worker value with either of the types:*

$$F' < F \implies \Pi(F, w(F, V)) > \Pi(F', w(F', V)) \text{ if } V = \bar{V}(F') \text{ or } V = \bar{V}(F). \quad (29)$$

Proof. This proof uses that the turnover is higher for a match of quality F' than for F , and that these quits are inefficient. Before renegotiation, the turnover in both jobs are the same, as worker values are the same (the wage is higher in the match with lower quality to compensate for a lower wage after renegotiation). However, after renegotiation, both workers receive their equilibrium values $\bar{V}(F)$ and $\bar{V}(F')$, respectively. Since $\bar{V}(F) > \bar{V}(F')$, any offer from a match of quality $\tilde{F} \in (F', F)$ will be accepted by a worker in a match with quality F' , but not by a worker in a match with quality F . This implies that the quit rate is strictly higher for F' than for F . Since the profits are strictly positive, a lower turnover increases total match surplus. Since worker values are the same at the beginning of the match, the profits (i.e. the firm value of the match) are therefore greater for F than for F' . ■

Lemma 2 *If the wage function $w(F)$ satisfies the differential equation (27), then the Nash product $V(F, w)^\beta \Pi(F, w)^{1-\beta}$, for a fixed match quality F , is strictly increase as a function of the wage for $w < w(F)$, and strictly decreasing for $w > w(F)$, provided that there exists a type F' , such that $w = w(F, \bar{V}(F'))$.*

Proof. The condition that $w = w(F, \bar{V}(F'))$ for some F' allows me to connect each wage to a worker value associated with a particular match quality F' . Later, it will be shown that all wages in the interior of the range of the wage function satisfies this condition.

Consider the derivative of the (log) Nash product at the wage w , where w satisfies $w = w(F, \bar{V}(F'))$. This derivative is given by

$$D(F, F') = \left[\frac{\beta}{\bar{V}(F')} \frac{\partial V(F, w(F, \bar{V}(F')))}{\partial w} \Big|_{w=w(F, \bar{V}(F'))} + \frac{1-\beta}{\Pi(F, w(F, \bar{V}(F')))} \frac{\partial \Pi(F, w(F, \bar{V}(F')))}{\partial w} \Big|_{w=w(F, \bar{V}(F'))} \right]$$

(recall that $V(F, w(F, V')) = V'$ for all V'). By the definition of the differential equation (27), we have $D(F, F) = 0$, i.e. the value is zero when $F = F'$.

Dividing $D(F, F')$ by the derivative of the worker value function and substituting for the derivative of the worker and firm value functions, we obtain

$$\frac{D(F, F')}{\frac{\partial V(F, w(F, \bar{V}(F')))}{\partial w} \Big|_{w=w(F, \bar{V}(F'))}} = \left[\frac{\beta}{\bar{V}(F')} - (1-\beta) \left[\frac{1}{\Pi(F, w(F, \bar{V}(F')))} - \frac{\delta + \rho + \lambda_e(1-F')}{\delta + \rho + \gamma(F) + \lambda_e(1-F')} \frac{\lambda_e}{w'(F')} \right] \right]. \quad (30)$$

Now, suppose that $w < w(F)$. Since $w = w(F, \bar{V}(F'))$, this means that $F' < F$, as the wage function is strictly increasing in the match quality. I want to show that $D(F, F') > 0$, and in order to do so, I use that $D(F', F') = 0$ and that $D(F, F') > D(F', F')$ when $F > F'$. Indeed, note that Lemma 1 gives us that $\Pi(F, w(F, \bar{V}(F'))) > \bar{\Pi}(F')$. Furthermore, $\gamma(F) \leq \gamma(F')$. This means that

the whole expression in square brackets is smaller for F , which implies that $D(F, F') > D(F', F') = 0$. By analogous reasoning, $D(F, F') < 0$ whenever $F' > F$.

■

Lemma 3 *If the wage function satisfies the differential equation (27), the equilibrium wage attains a global maximum of the Nash product for all types F if and only if the initial condition for the differential equation is (28).*

Proof. In order to prove the equilibrium existence-statement in Proposition 1, we only need the if-statement. However, the only-if statement will be used later to prove uniqueness.

I want to show that the Nash product is maximized at the bargaining outcome. This means that I seek to exclude that any other pre-renegotiation wages can increase the Nash product (we only need to consider this set of deviations, as the wage reverts back to the equilibrium bargaining outcome after renegotiation). As in Lemma 2, I formulate the problems in terms of worker value. Indeed, provided that the firm can do negative wage payments, any worker value can be attained by changing the pre-renegotiation wage.

In Lemma 2, we noted that the Nash product could not be increased by deviating to a worker value that was attained in equilibrium. When the wage function satisfies (27), the equilibrium set of worker values is connected, so the only possible deviations consist in going above or below the set of equilibrium worker values.

Given that we are interested in the behavior of the Nash product outside the set of equilibrium worker values, where the turnover effect is zero, it is useful to define the worker value that sets the derivative of the Nash product to zero when we treat turnover as fixed. We will do this for two cases. Firstly, the case when there is no endogenous quitting before the renegotiation is denoted with a subscript $+$. Secondly the case when the worker leaves for all jobs prior to renegotiation is denoted with a $-$. The Nash product for the first and second case is maximized at the worker value that solves

$$\frac{\beta}{\bar{V}_+(F)} - \frac{(1-\beta)}{\Pi(F, w(F, V_+(F)))} = 0 \quad \text{and} \quad \frac{\beta}{\bar{V}_-(F)} - \frac{(1-\beta)}{\Pi(F, w(F, V_-(F)))} = 0. \quad (31)$$

The Nash product is increasing prior to $\bar{V}_+(F)$ and decreasing thereafter if $\bar{V}_+(F)$ is above the support and analogously for $\bar{V}_-(F)$. Firstly, if it was the fact that $\bar{V}_-(0) < \bar{V}(0)$ the the Nash product increases as the wage is lowered below the support contradicting that $\bar{V}(0)$ attains the maximum. Secondly, assume that $\bar{V}_-(0) > \bar{V}(0)$. Recall that the derivative of the log Nash product,

(30) at worker value V , contains a third (weakly) positive term coming from the endogenously lower turnover. $\bar{V}_-(0) > \bar{V}(0)$ can therefore not be a maximum. Note further that the differential equation implies that wages increase quicker with match quality. This implies that $\bar{V}_-(0) < \bar{V}_-(F) \leq \bar{V}(F)$. Hence, no firm has a maximum below the support. Lastly $\bar{V}_+(F) < \bar{V}_+(1) \leq \bar{V}(1)$ and the maximum can therefore not occur above the support either.

■

Lemma 4 *If the profit function is locally strictly decreasing in the wage around the bargaining outcome, then the offers satisfy*

$$V(F, w_{f,\Delta}(F)) = (1 - \Delta)^{(1-\beta)} V(F, w_{w,\Delta}(F)) \quad (32)$$

$$\Pi(F, w_{w,\Delta}(F)) = (1 - \Delta)^\beta \Pi(F, w_{f,\Delta}(F)). \quad (33)$$

Proof. Since the worker's value function is globally strictly increasing in the wage, the worker will offer the highest wage such that the firm is indifferent between accepting the offer this period and rejecting and making a counter-offer next period. The profit function is weakly decreasing and continuous and there therefore there exist and unique highest wage. Indeed, the set is of offers that yield higher continuation value to the firm is closed as Π is continuous. Thus, it has a unique largest element. This gives the necessary conditions

$$V(F, w_{f,\Delta}(F)) = (1 - \Delta)^{(1-\beta)} V(F, w_{w,\Delta}(F)) \quad (34)$$

$$\Pi(F, w_{w,\Delta}(F)) = (1 - \Delta)^\beta \Pi(F, w_{f,\Delta}(F)). \quad (35)$$

These equations imply that the worker is indifferent between accepting an offer by the firm and rejecting it and waiting one period, similarly for the firm. If there were an inequality then an offer that is ϵ higher (lower) by the worker (firm) would be accepted since the worker's value function and firm's profit function are continuous. As the worker's (firm's) value function is increasing (decreasing), such an offer is preferred. ■

With these Lemmas, we can proceed to prove the proposition that any solution to the differential equation (27) with initial condition 28 constitutes an equilibrium.

Lemma 5 *There exist two offer that converge to the bargaining outcome as the friction goes to zero.*

Proof. This proof will be constructive. We will find that for sufficiently small Δ we can find a sequence of offers for the worker and firm that converge to the bargaining outcome. First note that

the (27) implies that the profit function is differentiable and decreasing in the wage evaluated at any weakly higher wage. Take the equilibrium from Lemma 4 and combining (32) and (33) we see that they imply that

$$V(F, w_{f,\Delta}(F))^\beta \Pi(F, w_{f,\Delta}(F))^{1-\beta} = V(F, w_{w,\Delta}(F))^\beta \Pi(F, w_{w,\Delta}(F))^{1-\beta}$$

The Nash product must therefore be the same when evaluated at the offer by the worker and the firm. For all F , an offer by the worker and the firm, around the bargaining outcome $w(F)$, can then be found that satisfies. The two offer must converge to the maximum of the Nash product which occurs at the bargaining outcome $w(F)$, see Lemma 2.

■

Hence, an equilibrium exists.

■

Proposition 2 *In all equilibrium, on the interior support, the wage function satisfies (27).*

Proof. The proof proceeds using a number of Lemmas. First, I prove that in any bargaining outcome both agents get a positive payoff.

Lemma 6 *For any $\Delta \in (1, 0)$ neither $\bar{x}(F)$ nor $\bar{b}(F)$ is a bargaining outcome.*

Proof. The highest possible continuation value for the worker matched to a match of quality F is the value function associated with the wage of $\bar{x}(F)$. The value function for the worker is differentiable in the wage. This means that, for any Δ , there exists an ϵ such that an offer of $\bar{x}(F) - \epsilon$ is strictly preferred by the worker compared to waiting one period to get $\bar{x}(F)$. Such an offer will be accepted by the worker since the assumption of Markov strategies imply that agents will accept any offer that results in a higher value than the continuation payoff. Thus, the firm will not accept an offer of $\bar{x}(F)$, as the firm would get a strictly positive payoff by rejecting and then offering $\bar{x}(F) - \epsilon$ in the next period. The analogous argument applies to $\bar{b}(F)$. ■

Lemma 7 *In equilibrium, all offers are accepted. Furthermore, the worker's and the firm's offers must converge as $\Delta \rightarrow 0$.*

Proof. First, I exclude the possibility that no agreement is ever reached. If no agreement is reached, then for all Δ , there exist some $\epsilon > 0$ such that an offer of $\bar{b}(F) + \epsilon$ by the firm must be accepted by the worker. (Note that, $\bar{b}(F)$ is equal to $= \bar{b}$ if there is no agreement, as that is the

outside value of the worker). The firm then has a profitable deviation, as such an offer would yield positive profits to the firm (given the assumption $x(0) > \bar{b}$). Thus, an agreement must be reached for all F .

Suppose that the equilibrium features agreement at a wage w . Then, the proposer when agreement is reached would accept w as the continuation value is lower given the cost of delay. This means that it cannot be optimal for the responder to offer an unacceptable wage, since this would be dominated by offering w . Thus, all offers are accepted. The worker's value function is strictly increasing in the wage, which implies that, if the two offers do not converge as the probability of breakdown disappears, the worker would reject the lower offer. Declining the lower offer would imply that there is not agreement after both offers. ■

Lemma 8 *In any equilibrium, the profit function is at the bargaining outcome to the right strictly decreasing in the wage.*

Proof. We only need to show that the profit function is strictly decreasing in worker's offers. I show this by contradiction. Suppose that the equilibrium features a firm wage offer w^f such that the profit function is locally non-decreasing to the right of w^f . Call the segment for which this is the case $[w^f, w^f + \delta)$, and let Π be the firm profit associated with wages in this range. Now, we know that the profit function is decreasing for sufficiently high wages, as the turnover effect disappears outside the support which. Given that the profit function is continuous, it is possible to choose a sufficiently high wage $w > w^f + \delta$ that yields profits $\Pi - \epsilon$ to the firm. For ϵ sufficiently small, such an offer will be acceptable. The offer by the worker must therefore always be at a point where the profit function is strictly decreasing in the wage. The bargaining outcome can therefore not lie on the interior of a nondecreasing part of the profit function. ■

Lemma 9 *If $w_{f,\Delta}(F) < w(F) < w_{w,\Delta}(F)$, then the profit function is strictly decreasing in the wage around the bargaining outcome.*

Proof. The Lemma will be proved using a contradiction. In particular I show that if for a type F the profit function is flat around the bargaining outcome then a type just above the F will have a derivative of the profit function that is close to zero. This implies from the bargaining equation that the derivative of the worker value function must also be close to zero, but this leads to a contradiction as the derivative of the worker value function is bounded below.

It is useful to first note that, it cannot be that the profit function is increasing in the wage at either the offer by the worker and the firm. If the profit function was strictly increasing in the wage

at an offer, both the firm and the worker would gain from a higher wage. A higher wage must then be accepted and is preferred by both parties. Such an offer is also preferable and the lower offer can therefore not constitute a Markov SPE. One implication of this is that there cannot be any mass points in the distribution. The reason is that a mass point means that the turnover effect locally dominates the increased cost for the firm and the profit function would therefore be increasing in the wage.

We now prove that there cannot be an equilibrium with a profit function that is constant in some region. Suppose that there exist a type \bar{F} such that the profit function is constant as the wage for some region. By Lemma 8, we know that the profit function is strictly decreasing to the right at the bargaining outcome. We can therefore restrict our attention to an interval below $w(\bar{F})$ which we denote by $[\underline{w}(\bar{F}), w(\bar{F})]$. Given that there is a continuous one-to-one mapping between the wage and the value function for the worker for a given match, we can define the interval of worker values this corresponds to. We denote $[V_u, V(\bar{F})]$ to be the range of worker values where the profit is constant for the match of quality \bar{F} .

We prove the contradiction in two steps. First we show that the function are wage and profit function are continuous. Then we show that this implies a contradiction for a type $\bar{F} - \epsilon$. The wage $w(F, V)$ is given by

$$(\delta + \rho + \lambda_e + \gamma(F))(V - \bar{V}(F)) = w(F, V) - w(F) + \lambda_e \int_0^1 \left(\max\{V, \bar{V}(\tilde{F})\} - \max\{\bar{V}(F), \bar{V}(\tilde{F})\} \right) d\tilde{F}.$$

The functions $\{\gamma(F), w(F), \bar{V}(F)\}$ are continuous in F and therefore so is $w(F, V)$. The profit function is given by

$$(\delta + \rho + \gamma(F) + \lambda_e(1 - G(V)))\Pi(F, w(F, V)) = x(F) - w(F, V) + \gamma(F)\Pi(F, w(F)).$$

The profit function must then be continuous in F , as all other functions in the expression are continuous in F . The profit function is differentiable in the region $(V_u, V(F))$ and the derivative at V is given by⁷

$$\frac{\partial \Pi(F, w)}{\partial w} \Big|_{w=w(F, V)} = -\frac{1}{\delta + \rho + \gamma(F) + \lambda_e(1 - G(V))} + \frac{\lambda_e G'(V)\Pi(F, w(F, V))}{(\delta + \rho + \gamma(F) + \lambda_e(1 - G(V)))^2}. \quad (36)$$

Where, we for the match \bar{F} , the effect of decreased turnover exactly matches the increased cost for a match \bar{F} , so that the derivative is zero. For the turnover effect to be positive it has to be that the profit function $\Pi(F, w(F))$ is strictly positive.

⁷Since the function $G(V)$ is differentiable in the region, $(V_u, V(\bar{F}))$, the profit function for any other match quality is also differentiable in this region of worker values.

Take a type $\bar{F} - \epsilon$ where the worker value associated with the bargaining outcome lies in the interval $(V_u, V(\bar{F}))$. By Lemma 8, the right derivative with respect to the wage is negative. The profit function must then be strictly decreasing in the wage and by Lemma 4, the bargaining equations, ((32) and (33)), must then hold with equality.

Since the derivative of the profit function is continuous in F , see (36), this implies that the derivative of profit function with respect to the wage for a type $\epsilon \rightarrow 0$ is zero. Thus for the bargaining equation (32) and (33) to hold, it has to be that the worker's value function is constant as $\epsilon \rightarrow 0$. But the derivative of the worker's value function is bounded below by $1/(\delta + \rho + \gamma(\bar{F} - \epsilon) + \lambda_e)$. The bargaining equation can therefore not hold for $\bar{F} - \epsilon$ and we can therefore reject that the profit function is flat in some part below the wage outcome.

■

It must then be that profit function is strictly decreasing in the wage in the region of wages where offers are made. Equation (32) and (33) must therefore hold by Lemma 4.

Lemma 10 *It must be that $w_{f,\Delta}(F) < w(F) < w_{w,\Delta}(F)$.*

Proof. We proceed by contradiction and assume $w_{f,\Delta}(F) < w_{w,\Delta}(F) \leq w(F)$. We know (32) and (33) hold with equality this implies that the Nash product must be the same at the offer by the firm and the worker. Taking the limit at the friction goes away implies that the Nash product must be constant, for type F , below the bargaining outcome. For a match $F - \epsilon$ the Nash product must also be differentiable. For this lower type the profits are lower, by Lemma 1, when evaluated at the same worker value. The derivative of the Nash product can therefore not be zero for two different types, see Lemma 2. This implies that two offers satisfying (32) and (33) that converge to $w(F - \epsilon)$ can not be found and $w(F - \epsilon)$ can then not be a bargaining outcome. The case $w(F) \leq w_{f,\Delta}(F) < w_{w,\Delta}(F)$ follow from the same argument. ■

Lemma 11 *The value functions are differentiable on the interior support.*

Proof. Rewriting (32) and (33) we get

$$\lim_{\Delta \rightarrow 0} \frac{V(F, w_{f,\Delta}(F))^{1/(1-\beta)} - V(F, w_{w,\Delta}(F))^{1/(1-\beta)}}{\Delta} = -V(F, w(F))^{1/(1-\beta)} \quad (37)$$

$$\lim_{\Delta \rightarrow 0} \frac{\Pi(F, w_{w,\Delta}(F))^{1/\beta} - \Pi(F, w_{f,\Delta}(F))^{1/\beta}}{\Delta} = -\Pi(F, w(F))^{1/\beta}. \quad (38)$$

We know that $w_{f,\Delta}(F) < w(F) < w_{w,\Delta}(F)$. This implies that the value function has a right and left derivative at $w(F)$. By Theorem 17.9 in Hewitt and Stromberg (2013) the left and right

derivative can at most differ countable many times if both exist everywhere in an open set. This implies that the derivatives exist almost everywhere. But that also implies that the left and right derivative are equal everywhere, on the interior support. At any point, the derivative on the right and left both have to satisfy the same differential equation, ((37) and (38)), and since the two value functions are continuous, the derivatives must then also be the same. ■

This implies that we can take the limit of the bargaining equations to get the differential equation (27).

■

The proof is presented in two steps. First I show that the Nash product is increasing and then decreasing on the support of values. I then show that the Nash product is only decreasing outside the support if (28) holds.

Proposition 3 *The lowest wage is an equilibrium if and only if the wage function satisfies (27) and (28).*

Proof.

We know, by equation (32) and (33), that the offer by the worker and the firm must imply the same Nash product. On the interior of the support, for the lowest type, the Nash product decreases as wage is raised. In order to find an offer around the lowest wage we therefore require that the Nash product decreases as the wage is lowered outside the support.

Lemma 12 *The bargaining outcome is weakly greater than the bargaining outcome that treats turnover as fixed.*

Proof. The derivative of the Nash product is

$$\frac{\beta}{\bar{V}(F)} - (1 - \beta) \left[\frac{1}{\Pi(F, w(F, \bar{V}(F)))} - \frac{\delta + \rho + \lambda_e(1 - F)}{\delta + \rho + \gamma(F) + \lambda_e(1 - F)} \frac{\lambda_e}{w'(F)} \right].$$

The effect of OJS is captured in the term

$$\frac{\delta + \rho + \lambda_e(1 - F)}{\delta + \rho + \gamma(F) + \lambda_e(1 - F)} \frac{\lambda_e}{w'(F)} \Pi(F, w(F, \bar{V}(F'))),$$

which reflects how the joint surplus changes with the wage. The lower turnover associated with a higher wage implies that the derivative of the Nash product is greater. As the derivative of Nash product is zero at the wage outcome, the bargaining outcome is weakly greater than had turnover been treated as fixed. ■

Lemma 13 *The Nash product decreases as the wage decreases or increases out of the support of wages if and only if*

$$\beta\Pi(0, w(0, \bar{V}(0))) = (1 - \beta)\bar{V}(0). \quad (39)$$

Proof. (39) solves for the maximum of the Nash product outside the support. If the wage is higher than this wage, the Nash product increases as the wage is lowered. For the Nash product to be decreasing at the lowest wage, we require (39) to hold. By Lemma 12, we know that the wage outcome is weakly greater than the wage treating turnover as fixed. The Nash product must therefore decrease above the highest wage. ■

■

Proposition 4 *There is a unique bargaining outcome for each match quality.*

Proof. I now prove that there is a unique bargaining outcome for each type. On the interior support we know that the differential equation has to hold but by Lemma 2 if it holds for one type it cannot hold for another type. Hence, no wage that implies a worker value that is on the interior support can be a bargaining outcome. A bargaining outcome less than the lowest value is not a bargaining outcome due to Lemma 12. Similarly a bargaining outcome above the distribution is not a bargaining outcome because of the same Lemma.

■

A.2 Theorem 3

First note that no firms wants to change the contract length (either the derivative with respect to the contract length is zero or the derivative is negative and the wages are continuously renegotiated). The derivative of the Nash product satisfies

$$\beta \frac{1}{\bar{V}(F')} - \frac{(1 - \beta)}{\Pi(F, w(F, \bar{V}(F')))} + (1 - \beta) \frac{\lambda_e}{w'(F')} \frac{\delta + \rho + \lambda_e(1 - F')}{\delta + \rho + \gamma(F) + \lambda_e(1 - F')}. \quad (40)$$

We need to show that the Nash product is increasing prior to the bargaining outcome and decreasing thereafter. If there is continuous renegotiation it is clear that the Nash product increases prior to the bargaining outcome and decreases thereafter. Alternatively when wages are infrequently renegotiated we can use the first order condition from the firms optimal choice of contract length, $\lambda_e\Pi(F', w(F', V')) = w'(F)$. Taking the derivative, with respect to F , of the derivative of the

Nash product, and simplifying, gives

$$\frac{\Pi_F(F, w(F, V'))}{\Pi(F', w(F'))} - (\delta + \rho + \lambda_e(1 - F')) \left(\frac{\gamma'(F')}{(\delta + \rho + \gamma(F') + \lambda_e(1 - F'))^2} \right) \quad (41)$$

The derivative of the contract length can be expressed as

$$\begin{aligned} \gamma'(F) = & -\lambda_e \frac{\beta \Pi(F, w(F))}{(1 - \beta) V(F, w(F)) - \beta \Pi(F, w(F))} \\ & - \frac{(1 - \beta) / \beta (\delta + \rho + \lambda_e(1 - F))}{((1 - \beta) V(F, w(F)) / \beta / \Pi(F, w(F))) - 1)^2} \left(\frac{V_F(F, w(F))}{\Pi(F, w(F))} - \frac{V(F, w(F))}{\Pi(F, w(F))^2} \Pi_F(F, w(F)) \right). \end{aligned}$$

Using the derivative of $\gamma(F)$ we can express the change in the derivative as

$$\begin{aligned} \Pi_F(F, w(F, V')) \left(\frac{V(F, w(F)) - \beta / (1 - \beta) \Pi(F', w(F'))}{\Pi(F', w(F')) V(F, w(F))} \right) + \lambda_e \frac{\beta \Pi(F, w(F))}{(\delta + \rho + \lambda_e(1 - F'))} \frac{(1 - \beta) V(F, w(F)) - \beta \Pi(F, w(F))}{(1 - \beta) V(F, w(F))^2} \\ + \frac{\beta / (1 - \beta) \Pi(F, w(F))^2}{V(F, w(F))^2} \frac{V_F(F, w(F))}{\Pi(F, w(F))}. \end{aligned}$$

where all terms are positive. The Nash product is therefore increasing prior to the bargaining outcome and decreasing thereafter.